SOFTWARE USER GUIDE

Gas Turbine Engine Calculation (GTECalc)

(version 1.0.1)

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Contents

Nomenclature

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Operator	Definition
$\blacksquare_0$	ambient
$\blacksquare_{in}$	inlet
$\blacksquare_{fan}$	fan
$\blacksquare_c$	compressor
$\blacksquare_g$	combustion chamber
$\blacksquare_t$	turbine
$\blacksquare_{ab}$	afterburner
$\blacksquare_n$	nozzle

Latin letters

Symbol	Dimension	Definition
a	m/s	speed of sound
a_{cr}	m/s	critical speed of sound
c_n	m/s	exit speed from nozzle
C_{sp}	kg/Nh	specific fuel consumption
G_a	kg/s	air mass flow
H	m	altitude
k		adiabatic exponent
L_e		effective work of cycle
M		Mach number
M_f		flying speed
p_0	Pa	ambient pressure
p_i^*	Pa	total pressure at section i
P	N	thrust
P_{sp}	Ns/kg	specific thrust
q_f		relative fuel consumption
$q(\lambda)$		gas dynamic function

T_0	K	ambient temperature
T_i^*	K	total temperature at section i
R	$J/_{kgK}$	gas constant
V_f	$^m/_s$	flying speed

Greek letters

Symbol	Definition
α	air-fuel ratio
β_g	mass change coefficient
δ_c	air fraction for turbine cooling
η_c^*	efficiency of compressor
η_e	effective efficiency of cycle
η_f	flying efficiency
η_m	mechanical efficiency
η_t	total efficiency of cycle
η_t^*	efficiency of turbine
λ	normalized velocity
π_c^*	pressure ratio of compressor
π_d	disposable pressure
π_t^*	pressure ratio of turbine
$\pi(\lambda)$	gas dynamic function
σ_i	pressure losses in i
$\tau(\lambda)$	gas dynamic function
φ_n	nozzle speed coefficient

Introduction

The program Gas Turbine Engine Calculation (GTECalc) is designed for the preliminary evaluation of gas turbine engines performance. It is included the calculation blocks for turbojet, turbojet with afterburner (ab), turbofan, turbofan with afterburner (ab) and turboprop engines. The first order model described in [1, 2, 3] has been used.

The program GTECalc is distributed for free as "as is" without warranties of any kind. The user assumes all responsibility and risk for GTECalc usage.

The author would like to thank Prof. PhD Bakulev V. and Prof. Dr. Krilov B. who were inspiring him for a long time. This program would not be possible without their encouragement.

Chapter 1. Graphical User Interface (GUI)

The graphical interface of GTECalc is presented in Fig. 1.1.

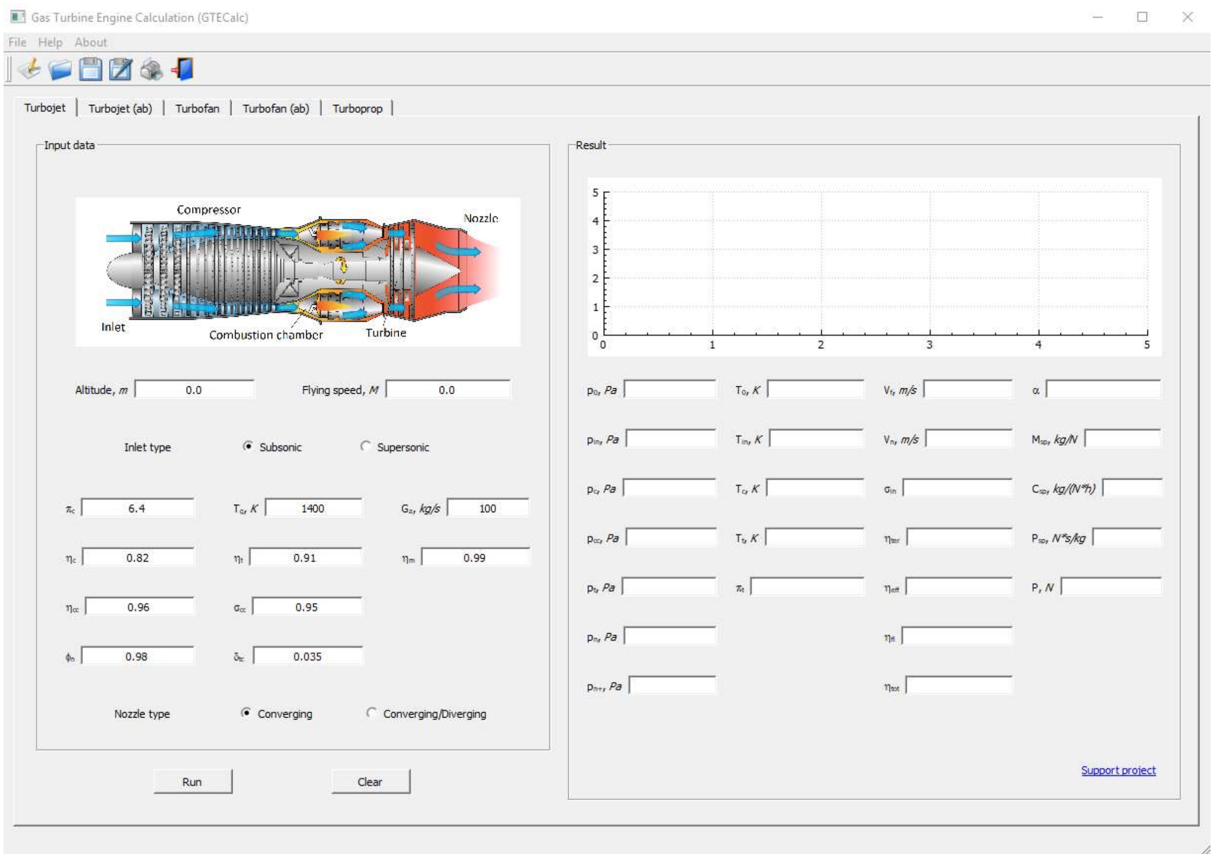

Fig. 1.1. The graphical interface of GTECalc.

The program has a Menu bar with general functions as New, Open, Save, Save As, Print, Exit, Help and About on the top. The functions from first menu are duplicated in the toolbar (Fig. 1.2). The functions can be called by combination of keys (hot keys). For example, function File can be called by combination Ctrl + F, function New by Ctrl + N, etc.

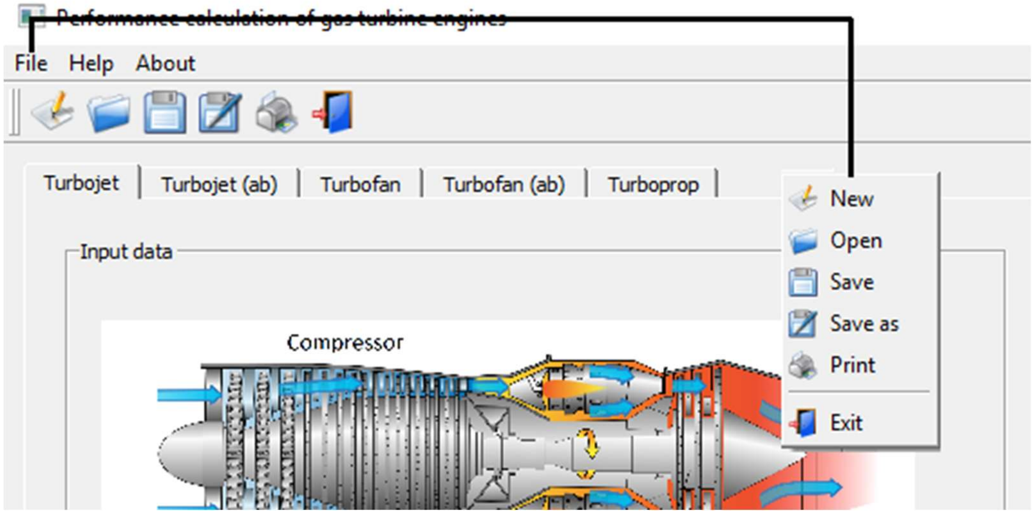

Fig. 1.2. Top menu and toolbar.

The working window has the tabs with different types of engines: turbojet, turbojet with afterburner (ab), turbofan, turbofan with afterburner (ab) and turboprop engines (Fig. 1.2). Each tab is divided to the two sections: left and right (Fig. 1.3).

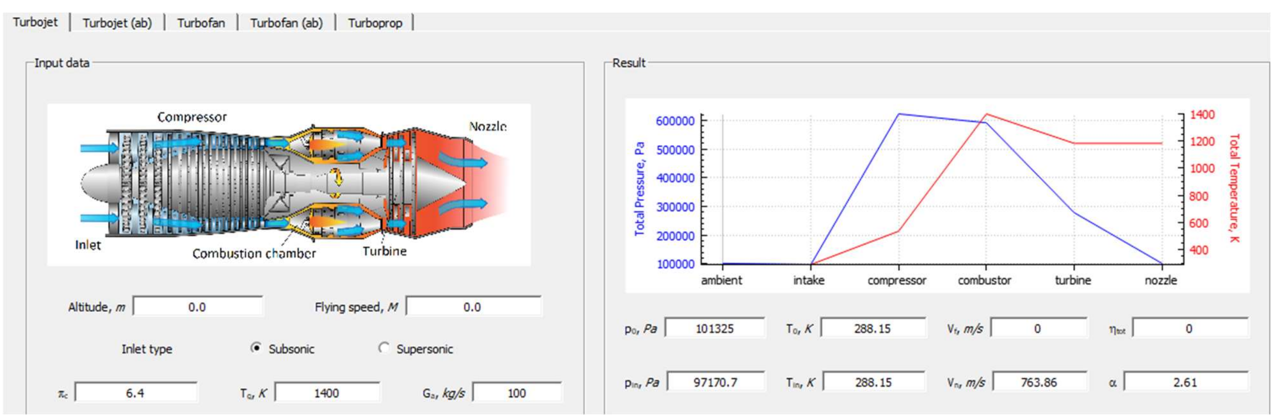

Fig. 1.3. Sections with input and result data.

On the left section there is a window with input data and scheme of engine. It has the input windows and several radio buttons to select the type of inlet, nozzle, etc. On the right section there is a window with the result(calculated) data. It has a graph on the top for the total pressure (blue color) and total temperature (red color) at the different engine sections and calculated data. By mouse pointing, the pop-up help with a parameter description can be called (Fig. 1.3).

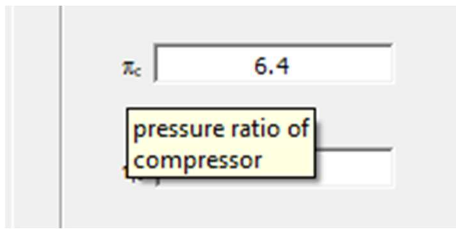

Fig. 1.4. Pop-up help.

The buttons for the calculation operation are situated on the bottom of working window (Fig. 1.5).

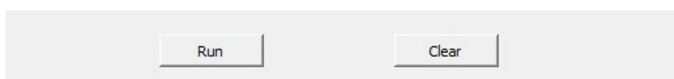

Fig. 1.5. Operation buttons.

The calculation can be started by “Run” button. The “Clear” button allows to clean input data. Similar to the menu functions instead of buttons the hot keys can be used, for example, button Run by Ctrl + R.

Chapter 2. Special and gas dynamic functions

The following special and gas dynamic functions have been used in the current software:

standard atmospheric conditions [6],

empirical pressure losses curves for subsonic and supersonic inlets [4],

empirical curves for enthalpies [1, 3],

gas dynamic functions $\pi(\lambda)$, $\tau(\lambda)$, $q(\lambda)$ [2].

To define the conditions at the different altitudes H the standard atmospheric conditions have been used [6]. The geopotential altitude is defined as

$$F = \frac{R_e H}{R_e + H}, \tag{2.1}$$

where $R_e = 6371210m$ is a radius of earth. The temperature of air is

$$T_0 = \begin{cases} T_{00} - \gamma F, 0 \leq F \leq 11000m \\ T_{11}, 11000 < F \leq 25000m \end{cases}, \tag{2.2}$$

where $T_{00} = 288.15K$ is a temperature on the sea level, $T_{11} = 216.66K$ is a temperature at altitude $H = 11000m$ and $\gamma = 0.00651122$ is a vertical temperature gradient. The atmospheric pressure is

$$P_0 = \begin{cases} P_{00}\left(1-\frac{\gamma F}{T_0}\right)^{\frac{g_0}{R\gamma}}, 0 \le F \le 11000m \\ P_{11}e^{-\frac{g(F-F_{11})}{RT}}, 11000 < F \le 25000m \end{cases}, \qquad (2.3)$$

where $P_{00} = 101325Pa$ is an atmospheric pressure on the sea level, P_{11} is an atmospheric pressure at altitude $H = 11000m$, $R = 287.053\frac{J}{kgK}$ is a gas constant and g_0 is an acceleration of gravity at altitude H. The acceleration of gravity can be defined as

$$g_0 = g_{00}\left(\frac{R_e}{R_e+H}\right)^2, \qquad (2.4)$$

where g_{00} is an acceleration of gravity on the sea level.

Empirical pressure losses curves for subsonic and supersonic inlets are extracted from Fig. 2.1 [4] by using Lagrange polynomial interpolation [5] with 7/9 points to provide smooth accurate curve approximation.

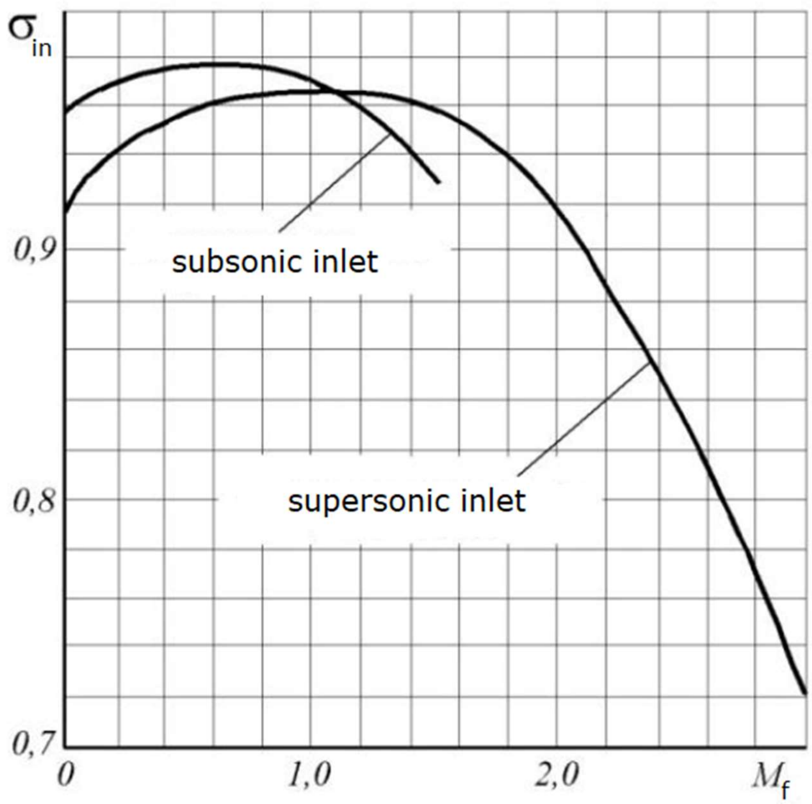

Fig. 2.1. Pressure losses in the inlets as a function of flying speed [4].

The empirical curves for enthalpies are extracted from Fig. 2.2-2.4 by using Lagrange polynomial interpolation [5] with 9 points.

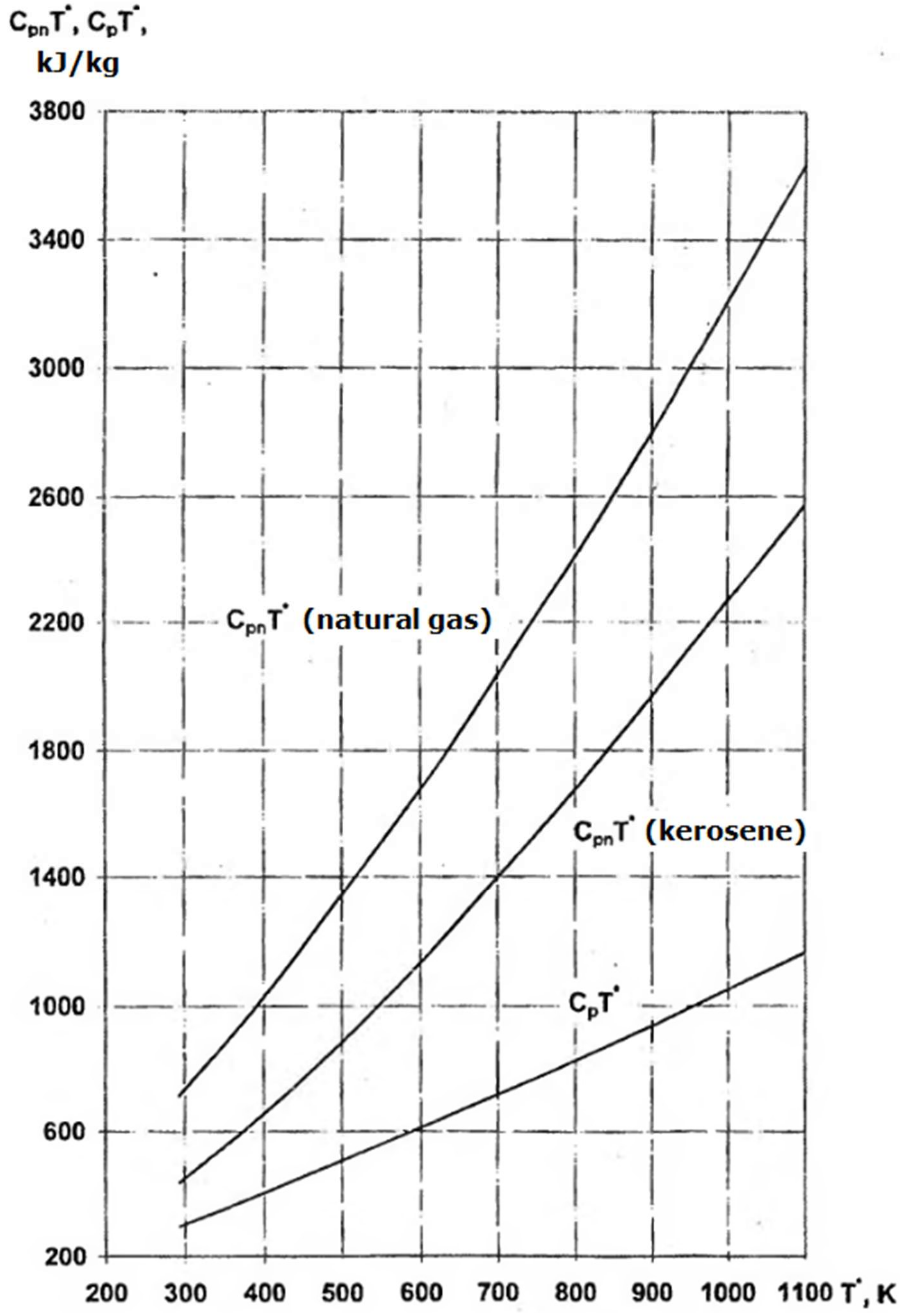

Fig. 2.2. Enthalpies as a function of combustion temperature (200K-1100K) [1].

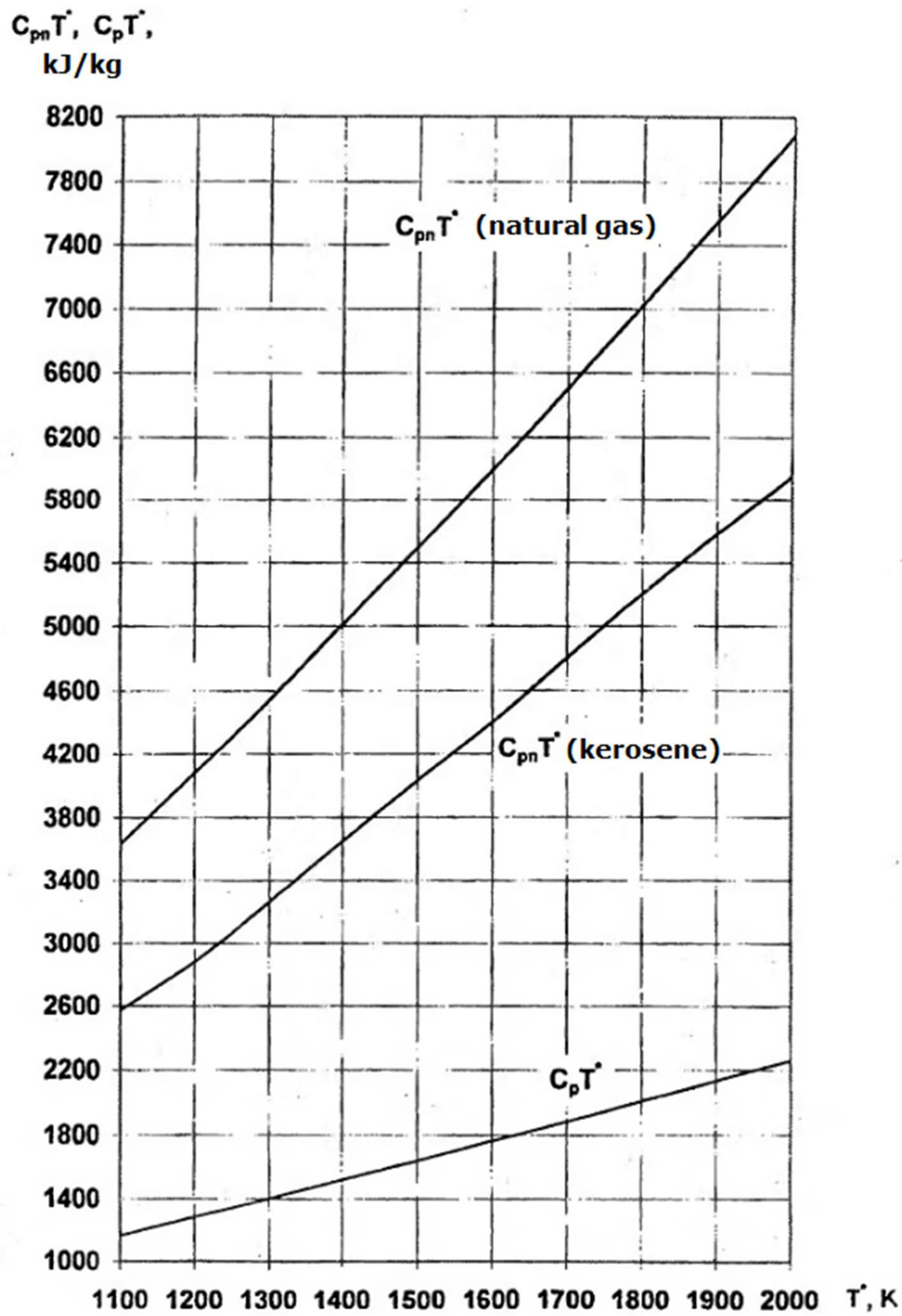

Fig. 2.3. Enthalpies as a function of combustion temperature (1100K-2000K) [1].

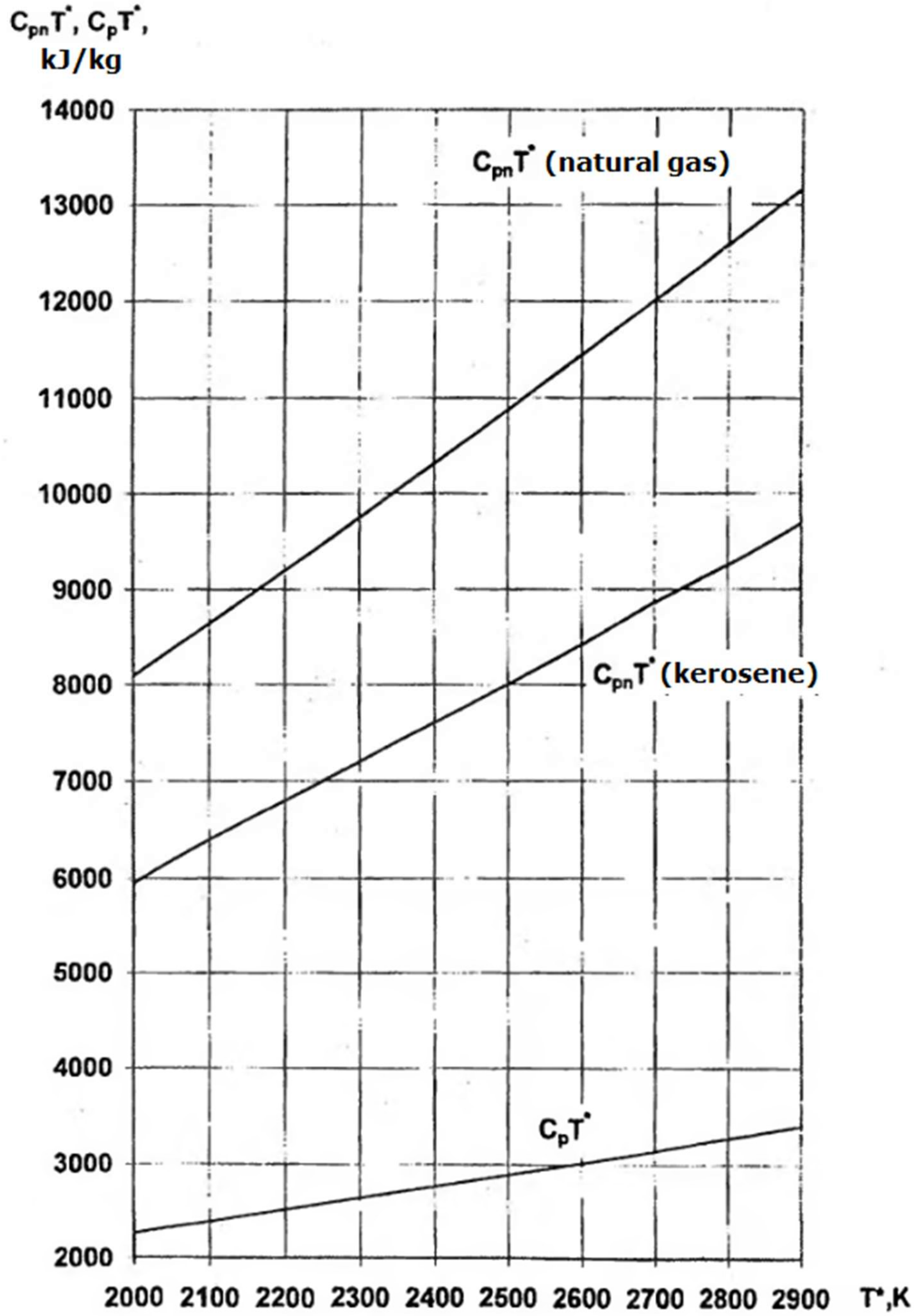

Fig. 2.4. Enthalpies as a function of combustion temperature (2000K-2900K) [1].

The gas dynamic function $\pi(\lambda)$ is calculated as

$$\pi(\lambda) = \left(1 - \frac{k-1}{k+1}\lambda^2\right)^{\frac{k}{k-1}} \tag{2.5}$$

$\tau(\lambda)$ as

$$\tau(\lambda) = 1 - \frac{k-1}{k+1}\lambda^2 \tag{2.6}$$

and $q(\lambda)$ as

$$q(\lambda) = \lambda\left[\left(1 - \frac{k-1}{k+1}\lambda^2\right)\left(\frac{k+1}{2}\right)\right]^{\frac{1}{k-1}}, \tag{2.7}$$

where λ is normalized velocity. The normalized velocity is defined as

$$\lambda = \frac{w}{a_{cr}}, \tag{2.8}$$

where w is a speed and a_{cr} is a critical speed of sound:

$$a_{cr} = \sqrt{\frac{2k}{k+1} RT^*}. \tag{2.9}$$

The gas dynamic functions can be calculated by using Mach number as well

$$M = \frac{w}{a}, \tag{2.10}$$

where a is a speed of sound:

$$a = \sqrt{kRT}. \tag{2.11}$$

The relation between Mach number and normalized velocity can be written as

$$\lambda^2 = \frac{\frac{k+1}{2} M^2}{1 + \frac{k-1}{2} M^2}. \tag{2.12}$$

Chapter 3. Turbojet engine

In general, an aircraft turbojet engine has inlet, compressor, combustion chamber turbine and nozzle (Fig. 3.1). The coming air is preliminary compressed in the inlet (subsonic/supersonic) if the aircraft is on fly. After it, the air is highly compressed in the axial compressor. The compressed air mix with fuel and burns in the combustion chamber. The burned gases are expanded in the turbine that rotates the compressor. The rest of energy is going to the nozzle to generate a jet thrust. The detailed description of turbojet engine working principles can be found in the specialized literature [1, 2, 3].

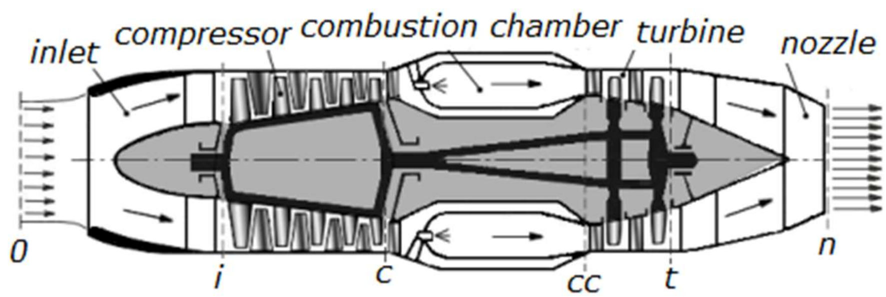

Fig. 3.1. Scheme of turbojet engine.

The calculation of turbojet engine is processed by sections: from the section before engine inlet to the end at the nozzle exit.

At the given altitude, the pressure p_0, temperature T_0 and speed of sound a_0 can be defined from standard atmospheric conditions.

1. The pressure losses σ_{in} in the supersonic or subsonic inlet are calculated based on the empirical curves [4]. The total pressure after inlet is

$$p_{in}^* = p_0\left(1+\frac{k-1}{2}M_f^2\right)^{\frac{k}{k-1}}\sigma_{in} = \frac{p_0\sigma_{in}}{\pi(\lambda_f)} \qquad (3.1)$$

and the total temperature is

$$T_{in}^* = T_0\left(1+\frac{k-1}{2}M_f^2\right) = \frac{T_0}{\tau(\lambda_f)}. \qquad (3.2)$$

The flying speed can be found as

$$V_f = M_f a_0. \qquad (3.3)$$

2. The total pressure after compressor is

$$p_c^* = p_{in}^* \pi_c^*. \qquad (3.4)$$

where π_c^* is air pressure ratio in the compressor.

The total temperature is

$$T_c^* = T_{in}^*\left(1+\frac{\pi_c^{*\frac{k-1}{k}}-1}{\eta_c}\right), \qquad (3.5)$$

where η_c is the efficiency of compressor.

3. The total pressure after combustion chamber is

$$p_g^* = p_c^* \sigma_g, \tag{3.6}$$

where σ_g is the pressure losses in the combustion chamber. The relative fuel consumption can be found from the following equation [1, 2, 3]

$$q_f = \frac{C_p T_g^* - C_p T_c^*}{H_u \eta_g - C_{p_n} T_g^* + C_{p_n} T_0}, \tag{3.7}$$

where the enthalpies $C_p T$ and $C_{p_n} T$ can be found from the empirical curves [1]. The lower heating value of kerosene is $H_u = 42900 \ \frac{kJ}{kg}$. The air-fuel ratio is

$$\alpha = \frac{1}{q_f L_0}, \tag{3.8}$$

where $L_0 = 14.8$ is the stochiometric coefficient for kerosene.

4. The pressure ratio in turbine π_t^* can be found from the balance equation for compressor and turbine. The final relation can be written as

$$\pi_t^* = \left[1 - \frac{k(k_g - 1) R T_{in}^*}{k_g (k-1) R_g T_g^*} \frac{1}{(1 + q_f)(1 - \delta_c)} \frac{\pi_c^{*\frac{k-1}{k}} - 1}{\eta_c \eta_t^* \eta_m}\right]^{\frac{k_g}{k_g - 1}}, \tag{3.9}$$

where η_t is an efficiency of turbine, η_m is a mechanical efficiency and δ_c is an air fraction required for turbine cooling.

The total pressure after turbine is

$$p_t^* = \frac{p_g^*}{\pi_t^*} \tag{3.10}$$

and the total temperature is

$$T_t^* = T_g^* \left[1 - \left(1 - \frac{1}{\pi_t^{*\frac{k_g - 1}{k_g}}} \eta_t^* \right) \right]. \tag{3.11}$$

5. The disposable pressure ratio at the nozzle is

$$\pi_{dn} = \frac{p_t^*}{p_0}. \tag{3.12}$$

If the pressure is fully discharged in the nozzle

$$\Delta p_n = p_n - p_0 = 0, \tag{3.13}$$

the normalized gas speed from the nozzle is

$$\lambda_n = \lambda_{ns} \varphi_n, \tag{3.14}$$

where a normalized isentropic gas speed from nozzle λ_{ns} can be find from

$$\pi(\lambda_{ns}) = \frac{p_0}{p_t^*} = \frac{1}{\pi_n}. \tag{3.15}$$

The total pressure at the nozzle exit is

$$p_n^* = \frac{p_0}{\pi(\lambda_n)}. \tag{3.16}$$

If the pressure is not fully discharged in the nozzle

$$\Delta p_n = p_n - p_0 \neq 0, \tag{3.17}$$

the normalized gas speed from nozzle is $\lambda_{ns} = 1$ and the pressure at the nozzle exit is

$$p_n = p_t^* * \pi(1) = 0.54 p_t^*. \tag{3.18}$$

The normalized gas speed from nozzle is

$$\lambda_n = \varphi_n. \tag{3.19}$$

and the total pressure at the nozzle exit is

$$p_n^* = \frac{p_n}{\pi(\lambda_n)}. \quad (3.20)$$

The exit speed from nozzle is

$$C_n = \lambda_n \sqrt{2\frac{k_g}{k_g+1} R_g T_t^*}. \quad (3.21)$$

6. The specific thrust is

$$P_{sp} = \big(1 + q_t(1-\delta_c)\big)C_n - V_f + \frac{\big(1+q_t(1-\delta_c)\big)\Delta p_n \sqrt{T_t^*}}{mq(\lambda_n)p_n^*}. \quad (3.22)$$

where a coefficient m is calculated as

$$m = \sqrt{\frac{1}{R} k_g \left(\frac{2}{k_g+1}\right)^{\frac{k_g+1}{k_g-1}}}. \quad (3.23)$$

The specific fuel consumption is

$$\mathrm{C}_{sp} = \frac{3600 q_f (1-\delta_c)}{P_{sp}}. \quad (3.24)$$

The thrust is

$$P = G_a P_{sp}. \tag{3.25}$$

where G_a is an air mass flow.

7. The effective efficiency of cycle is

$$\eta_e = \frac{L_e}{1000 q_f (1 - \delta_c) H_u}, \tag{3.26}$$

where L_e is an effective work of cycle that can be found from

$$L_e = \frac{C_n^2 q_f \left(1 + q_f (1 - \delta_c)\right) - V_f^2}{2}. \tag{3.27}$$

The flying efficiency is

$$\eta_f = \frac{P_{sp} V_f}{L_e}. \tag{3.28}$$

The total efficiency is

$$\eta_t = \eta_e \eta_f. \tag{3.29}$$

The thermal efficiency of cycle can be found from

$$\eta_{ter} = 1 - \frac{1}{\left(\frac{p_c^*}{p_0}\right)^{\frac{k-1}{k}}}. \tag{3.30}$$

Chapter 4. Turbojet engine with afterburner

The power of turbojet engine can be increased by installation of afterburner between turbine and nozzle. The afterburner provides the additional energy to the gases before nozzle. It increases the exit gases speed from nozzle and thrust. Usually, turbojet engines with afterburner are supplied with converging/diverging nozzle as such aircrafts are designed for supersonic flights. The turbojet engine with afterburner (ab) is shown in Fig. 4.1.

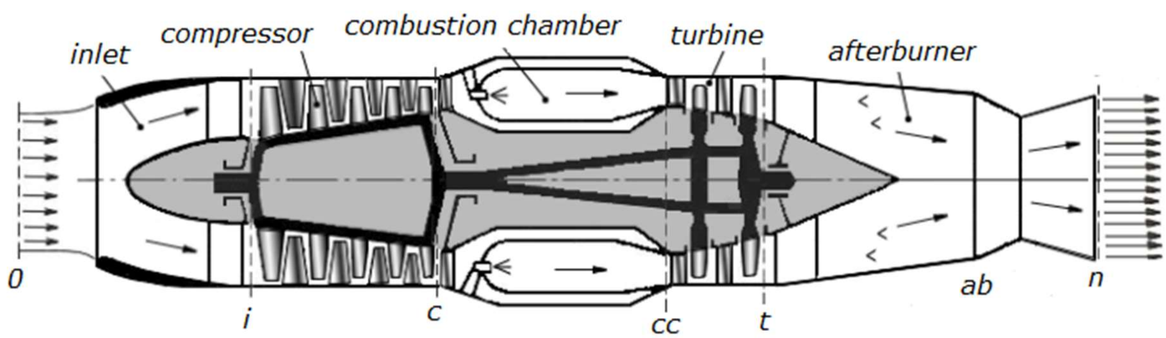

Fig. 4.1. Scheme of turbojet engine with afterburner.

The calculation of turbojet engine with afterburner is identical to the regular turbojet engine until a turbine section (Chapter 3(4)).

4 (ab). The total pressure after afterburner is

$$p_{ab}^{*} = p_{t}^{*}\sigma_{ab}, \tag{4.1}$$

where σ_{ab} is the pressure losses in the afterburner.

The relative fuel consumption in the afterburner is

$$q_{fab} = \frac{C_p T_{ab}^* - C_p T_t^* + q_f\left(C_{p_n} T_{ab}^* - C_{p_n} T_t^*\right)}{H_u \eta_{ab} - C_{p_n} T_{ab}^* + C_{p_n} T_0}, \tag{4.2}$$

where the enthalpies $C_p T$ and $C_{p_n} T$ can be found from the empirical curves [1]. The lower heating value of kerosene is $H_u = 42900 \frac{kJ}{kg}$.

5. The disposable pressure ratio at the nozzle is

$$\pi_{dn} = \frac{p_{ab}^*}{p_0}. \tag{4.3}$$

If the pressure is fully discharged in the nozzle the normalized gas speed from nozzle is

$$C_n = \varphi_n \sqrt{2 \frac{k_g}{k_g - 1} R_g T_{ab}^* \left(1 - \frac{1}{\pi_{dn}^{\frac{k_g - 1}{k_g}}}\right)} \tag{4.4}$$

or by using gas dynamic function:

$$C_n = \lambda_n \sqrt{2 \frac{k_g}{k_g + 1} R_g T_{ab}^*}. \tag{4.5}$$

6. The specific thrust is

$$P_{spab} = \left(1 + q_f(1 - \delta_c) + q_{fab}\right)C_n - V_f \tag{4.6}$$

and the specific fuel consumption is

$$\mathrm{C}_{sp} = \frac{3600(q_f(1 - \delta_c) + q_{fab})}{P_{sp}}. \tag{4.7}$$

The thrust is

$$P_{ab} = G_a P_{spab}. \tag{4.8}$$

7. The effective efficiency of cycle is

$$\eta_{eab} = \frac{L_{eab}}{q_f(1 - \delta_c) + 1000 q_{fab} H_u}, \tag{4.9}$$

where L_{eab} is an effective work of cycle that can be found from

$$L_{eab} = \frac{C_n^2\left(1 + q_f(1 - \delta_c) + q_{fab}\right) - V_f^2}{2}. \tag{4.10}$$

The flying efficiency is

$$\eta_f = \frac{P_{spab} V_f}{L_{eab}}. \tag{4.11}$$

The total efficiency is

$$\eta_t = \eta_{eab}\eta_f. \tag{4.12}$$

Chapter 5. Turbofan engine

Nowadays, the usage of turbojet engines is drastically reduced due to a low efficiency. To increase the efficiency, the additional bypass duct has been introduced (Fig. 5.1). The bypass air going through a fan directly to the separated nozzle. The ratio between bypass and core mass flow is called bypass ratio. Sometimes, the bypass air mix with core gases in the mixing section situated between turbine and nozzle. Such scheme usually is used with afterburner (Chapter6) or in the turbofan engines with low bypass ratio. Here, the turbofan engine with separated nozzle is considered (Fig. 5.1).

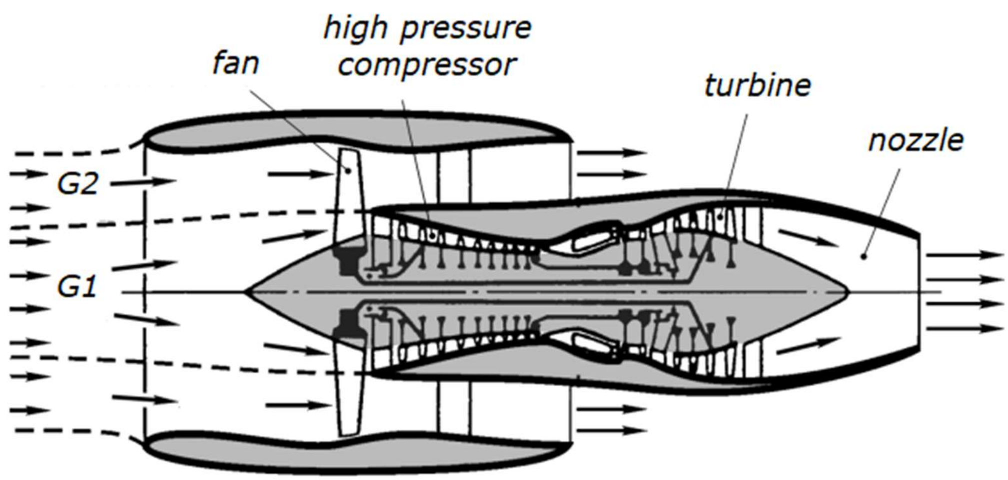

Fig. 5.1. Scheme of turbofan engine.

1. The calculation of inlet section is identical to the turbojet engine.

2 (fan). The total pressure after fan is

$$p_{fan}^{*} = p_{in}^{*}\pi_{fan}^{*}. \tag{5.1}$$

where π_{fan}^{*} is a pressure ratio of fan. The total temperature after fan is

$$T_{fan}^{*} = T_{in}\left(1 + \frac{\pi_{fan}^{\frac{k-1}{k}} - 1}{\eta_{fan}}\right), \tag{5.2}$$

where η_{fan} is an efficiency of fan.

2-4. The calculation of core (gas generator) is identical to the turbojet engines until a section after turbine of high-pressure compressor (HPC).
5. The pressure ratio of fan's turbine can be found as

$$\pi_{tfan}^{*} = \frac{1}{\left(1 - \mathrm{X}_{tfan}\right)^{\frac{k_g}{k_g - 1}}}, \tag{5.3}$$

where the coefficient X_{tfan} is

$$\mathrm{X}_{tfan} = \frac{C_p(1+m)(T_{in}^{*} - T_{fan}^{*})}{C_{pg}T_{tc}^{*}\eta_{tfan}^{*}(1+q_f)(1-\delta_c)\eta_{mfan}}. \tag{5.4}$$

Here T_{tc}^* is a total temperature after compressor's turbine, η_{tfan}^* is an efficiency of fan's turbine and η_{mfan} is a mechanical efficiency of fan's turbine.

The total pressure after turbine is

$$p_t^* = \frac{p_{tc}^*}{\pi_{tfan}^*}, \tag{5.5}$$

and the total temperature is

$$T_t^* = T_{tc}^*\left[1 - \mathrm{X}_{tfan}\eta_{tfan}^*\right]. \tag{5.6}$$

5 (int). The calculation of internal (core) exhaust is similar to the turbojet engine (Chapter 3). Below, the algorithm to calculate the core exhaust with converging nozzle is presented.

If the pressure is fully discharged in the nozzle the normalized gas speed from the nozzle can be calculated as

$$\lambda_{n1} = \lambda_{n1s}\varphi_{n1}. \tag{5.7}$$

The normalized isentropic gas speed from nozzle λ_{n1s} can be find from

$$\pi(\lambda_{n1s}) = \frac{p_0}{p_t^*} \tag{5.8}$$

and the total pressure at the nozzle exit of internal duct is

$$p_{n1}^{*} = \frac{p_0}{\pi(\lambda_{n1})}. \tag{5.9}$$

If the pressure is not fully discharged in the nozzle the normalized gas speed from nozzle is $\lambda_{n1s} = 1$ and the pressure at the nozzle exit is

$$p_{n1} = p_t^{*} * \pi(1) = 0.54 p_t^{*}. \tag{5.10}$$

The normalized gas speed from nozzle is

$$\lambda_{n1} = \varphi_{n1}. \tag{5.11}$$

and the total pressure at the nozzle exit is

$$p_{n1}^{*} = \frac{p_{n1}}{\pi(\lambda_{n1})}. \tag{5.12}$$

The exit speed from nozzle is

$$C_{n1} = \lambda_{n1}\sqrt{2\frac{k_g}{k_g+1}R_g T_t^{*}}. \tag{5.13}$$

The equivalent gas speed from the nozzle exit is

$$C'_{n1} = C_{n1} + \left[\frac{\sqrt{T_t^*}(p_{n1} - p_0)}{m_2 q(\lambda_{n1}) p_{n1}^*} \right]. \tag{5.14}$$

6 (int). The specific thrust of core is

$$P_{sp1} = \left(1 + q_f(1 - \delta_c)\right) C'_{n1} - V_f \tag{5.15}$$

5 (ext). The calculation of external (bypass) exhaust is similar to the internal exhaust.

The total pressure at the entrance of external nozzle is

$$p_2^* = p_{fan}^* * \sigma_2, \tag{5.16}$$

where σ_2 is the pressure losses in the external duct.

If the pressure is fully discharged in the nozzle the normalized gas speed from the nozzle can be calculated as

$$\lambda_{n2} = \lambda_{n2s} \varphi_{n2}. \tag{5.17}$$

The normalized isentropic gas speed from nozzle λ_{n1s} can be find from

$$\pi(\lambda_{n2s}) = \frac{p_0}{p_{n2}^*} \tag{5.18}$$

and the total pressure at the nozzle exit of external duct is

$$p_{n2}^{*} = \frac{p_0}{\pi(\lambda_{n2})}. \tag{5.19}$$

If the pressure is not fully discharged in the nozzle the normalized gas speed from nozzle is $\lambda_{n2s} = 1$ and the pressure at the nozzle exit is

$$p_{n2} = p_2^{*} * \pi(1) = 0.54 p_2^{*}. \tag{5.20}$$

The normalized gas speed from nozzle is

$$\lambda_{n1} = \varphi_{n1}. \tag{5.21}$$

and the total pressure at the nozzle exit is

$$p_{n1}^{*} = \frac{p_{n1}}{\pi(\lambda_{n1})}. \tag{5.22}$$

The exit speed from nozzle is

$$C_{n1} = \lambda_{n2} \sqrt{2 \frac{k}{k+1} R T_{fan}^{*}}. \tag{5.23}$$

The equivalent gas speed from the nozzle exit is

$$C'_{n2} = C_{n2} + \left[\frac{\sqrt{T^*_{fan}}(p_{n2} - p_0)}{m_2 q(\lambda_{n2}) p^*_{n2}}\right]. \tag{5.24}$$

6 (ext). The specific thrust of bypass duct is

$$P_{sp} \; = C'_{n2} - V_f. \tag{5.25}$$

6. The total specific thrust is

$$P_{sp} = \frac{P_{sp1} + mP_{sp}}{1 + m} \tag{5.26}$$

and the specific fuel consumption is

$$\mathrm{C}_{sp} = \frac{3600 q_f (1 - \delta_c)}{P_{sp}(1 + m)}. \tag{5.27}$$

The total thrust is

$$P = G_a P_{sp}. \tag{5.28}$$

7. The effective efficiency of cycle is

$$\eta_e = \frac{L_e}{q_f(1-\delta_c)1000H_u}, \tag{5.29}$$

where L_e is an effective work of cycle that can be found from

$$L_e = \frac{C_{n1}'^2\left(1 + q_f(1-\delta_c)\right) + mC_{n2}'^2 - (1+m)V_f^2}{2}. \tag{5.30}$$

The flying efficiency is

$$\eta_f = \frac{(1+m)P_{sp}V_f}{L_e}. \tag{5.31}$$

The total efficiency is

$$\eta_t = \eta_e\eta_f. \tag{5.32}$$

Chapter 6. Turbofan engine with afterburner

Similar to the turbojet engine, the afterburner can be installed in the turbofan engine to increase the maximal thrust. Usually the turbofan engine with afterburner has a mixing chamber and afterburner after it (Fig. 6.1).

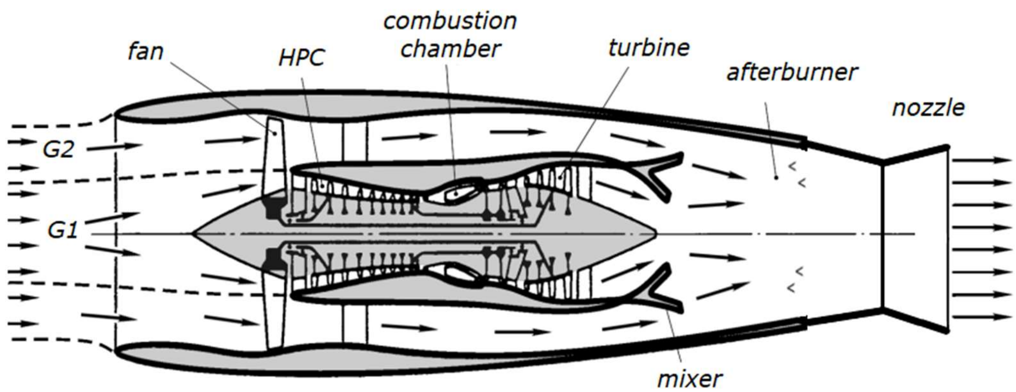

Fig. 6.1. Scheme of turbofan engine with afterburner.

The calculation of turbofan engine with afterburner is similar to turbofan engine for the section up to afterburner and to turbojet with afterburner for the section after afterburner.

4. The total pressure after turbine is

$$p_t^* = p_1^* = \frac{p_{tc}^*}{\pi_{tfan}^*}, \tag{6.1}$$

and the total temperature is

$$T_t^* = T_1^* = T_{tc}^*\left[1 - \mathrm{X}_{tfan}\eta_{tfan}^*\right]. \tag{6.2}$$

4 (mix). The pressure at entrance of mixing chamber from internal contour is

$$p_1 = p_1^*\pi(\lambda_1) = p_t^*\pi(\lambda_1). \tag{6.3}$$

The total pressure at the entrance of mixing chamber from external contour is

$$p_2^* = p_{fan}^*\sigma_2 \tag{6.4}$$

and the total temperature is

$$T_2^* = T_{fan}^*. \tag{6.5}$$

The normalized gas speed at the entrance of mixing chamber from external contour can be find based from condition that static pressure is equal for internal and external contour exit $p_1 = p_2$:

$$\pi(\lambda_{n2}) = \frac{p_2}{p_2^*} = \frac{p_1}{p_{fan}^*\sigma_2}. \tag{6.6}$$

The total temperature of mixed gas is

$$T_{mix}^{*} = \frac{T_t^{*}(1 + m\frac{T_{fan}^{*}}{T_t^{*}})}{1 + m}\,. \tag{6.7}$$

and the total pressure at the mixing chamber exit is

$$p_{mix}^{*} = \frac{\frac{p_1^{*}q(\lambda_1)}{q(\lambda_{mix})}\sqrt{\frac{T_{mix}^{*}}{T_t^{*}}}(1 + m)}{(1 + m\frac{p_1^{*}q(\lambda_1)}{p_2^{*}q(\lambda_2)})\sqrt{\frac{T_{fan}^{*}}{T_t^{*}}}}\,. \tag{6.8}$$

4 (ab). The total pressure after afterburner is

$$p_{ab}^{*} = p_t^{*}\sigma_{ab}. \tag{6.9}$$

The relative fuel consumption in the afterburner is

$$q_{fab} = \frac{C_pT_{ab}^{*} - C_pT_{mix}^{*} + \frac{q_f\left(C_{p_n}T_{ab}^{*} - C_{p_n}T_{mix}^{*}\right)}{1 + m}}{H_u\eta_{ab} - C_{p_n}T_{ab}^{*} + C_{p_n}T_0}, \tag{6.10}$$

where the enthalpies C_pT and $C_{p_n}T$ can be found from the empirical curves [1].

5. If the pressure is fully discharged in the nozzle the normalized gas speed from nozzle is

$$C_{nab} = \lambda_n \sqrt{2\frac{k_g}{k_g + 1} R_g T_{ab}^*}. \tag{6.11}$$

6. The specific thrust is

$$P_{spab} = \left(1 + \frac{q_f(1 - \delta_c)}{1 + m} + q_{fab}\right) C_{nab} - V_f \tag{6.12}$$

and the specific fuel consumption is

$$\mathrm{C}_{spab} = \frac{3600(\frac{q_f(1 - \delta_c)}{1 + m} + q_{fab})}{P_{spab}}. \tag{6.13}$$

The thrust is

$$P_{ab} = G_a P_{spab}. \tag{6.14}$$

7. The effective efficiency of cycle is

$$\eta_{eab} = \frac{L_{eab}}{1000H_u(\frac{q_f(1-\delta_c)}{1+m} + q_{fab})}, \tag{6.15}$$

where L_{eab} is an effective work of cycle that can be found from

$$L_{eab} = \frac{C_{nab}^2\left(1 + \frac{q_f(1-\delta_c)}{1+m} + q_{fab}\right) - V_f^2}{2}. \tag{6.16}$$

The flying efficiency is

$$\eta_f = \frac{P_{spab}V_f}{L_{eab}}. \tag{6.17}$$

The total efficiency is

$$\eta_t = \eta_{eab}\eta_f. \tag{6.18}$$

Chapter 7. Turboprop engine

The turbojet and turbofan have relative low performance at the low speed. In this case the turboprop engines can be used (Fig. 7.1). In comparison to the turbofan engine, the turboprop engine uses a propeller instead of fan. Sometimes, the propeller's turbine is called a free turbine. As the rotational speed of rotor is high the propeller is connected to the rotor by gearbox that reduces the rotational speed. The jet trust of turboprop usually is below 15% of total power. Most of the aircrafts with turboprop engines fly at speed not exceeding 800 km/h.

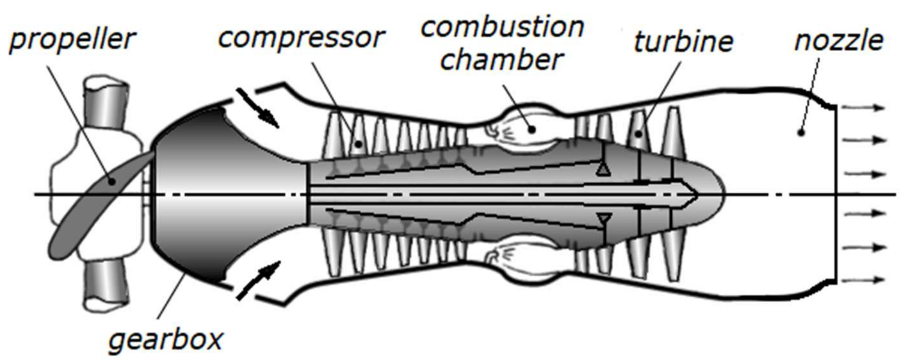

Fig. 7.1. Scheme of turboprop engine.

The calculation of turboprop engine is similar to the turbojet engine till section after compressor's turbine.

Several specific equations are introduced below. The free isentropic work is

$$L_{frs} = C_{pg} T_{tc}^{*} \left[1 - \left(\frac{p_0}{p_{tc}^{*}} \right)^{\frac{k-1}{k}} \right]. \qquad (7.1)$$

The optimal coefficient of free energy distribution between propeller and nozzle for $M_f > 0$ can be calculated from

$$\psi_{opt} = \frac{\varphi_n^2 V_f^2}{2L_{frs}\left(\eta_{tfr}^* \eta_p \eta_{pr}\right)^2}. \tag{7.2}$$

If $M_f = 0$, $\psi_{opt} = 0.008 \ldots 0.01$.

The isentropic nozzle work is

$$L_{ns} = L_{frs}\psi_{opt}, \tag{7.3}$$

and the isentropic work of free turbine is

$$L_{tfrs} = L_{frs}\left(1 - \psi_{opt}\right). \tag{7.4}$$

The work of free turbine is

$$L_{tfr} = L_{tfrs}\eta_{tfr}^*. \tag{7.5}$$

4 (frt). The pressure ratio of free turbine is

$$\pi_{tfr}^{*} = \frac{1}{\left(1 - \frac{\mathrm{L}_{tc}}{C_{pg} T_{tc}^{*}}\right)^{\frac{k_g}{k_g - 1}}}. \tag{7.6}$$

The total pressure after free turbine is

$$p_t^{*} = \frac{p_{tc}^{*}}{\pi_{tfr}^{*}}, \tag{7.7}$$

and the total temperature is

$$T_t^{*} = T_{tc}^{*} - \frac{L_{tfr}}{C_{pg}}. \tag{7.8}$$

5. If the pressure is fully discharged in the nozzle the normalized gas speed from the nozzle can be calculated as

$$\lambda_n = \lambda_{ns} \varphi_n. \tag{7.9}$$

The normalized isentropic gas speed from nozzle λ_{ns} can be find from

$$\pi(\lambda_{ns}) = \frac{p_0}{p_t^{*}} \tag{7.10}$$

and the total pressure at the nozzle exit is

$$p_n^* = \frac{p_0}{\pi(\lambda_n)}. \tag{7.11}$$

The gas speed from the nozzle is

$$C_n = \lambda_n \sqrt{2 \frac{k_g}{k_g + 1} R_g T_t^*}. \tag{7.12}$$

6. The specific propeller power is

$$\frac{L_{tfr} \eta_p (1 + q_f)(1 - \delta_c)}{1000}. \tag{7.13}$$

The specific equivalent power for $M_f > 0$ can be calculated from

$$N_{esp} = N_{psp} + \frac{V_f (C_n - V_f)(1 + q_f)(1 - \delta_c)}{1000}, \tag{7.14}$$

if $M_f = 0$:

$$N_{esp} = N_{psp} + 0.068(1 + q_f)(1 - \delta_c) C_n. \tag{7.15}$$

The specific fuel consumption is

$$\mathrm{C}_{esp} = \frac{3600 q_f}{N_{esp}}. \tag{7.16}$$

The equivalent engine power is

$$N_e = G_a N_{esp}. \tag{7.17}$$

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